

LIE THEORY AND ITS APPLICATIONS IN PHYSICS V

Varna, Bulgaria

16 – 22 June 2003

Organized by

Institute of Nuclear Research and Nuclear Energy
Bulgarian Academy of Sciences

Edited by

H.-D. Doebner
Technical University of Clausthal, Germany

V. K. Dobrev
University of Northumbria, UK
and
Bulgarian Academy of Sciences, Bulgaria

III. String Theory

- The Hagedorn/Deconfinement Phase Transition in Weakly Coupled Large N Gauge Theories 161
O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk
- Two-Loop Commuting Charges and the String/Gauge Duality 204
G. Arutyunov and M. Staudacher
- Boundary Ground Ring and Disc Correlation Functions in Liouville Quantum Gravity 217
I. Kostov
- Strings, Integrable Systems, Geometry and Statistical Models 231
A. Marshakov
- Impact of Dynamical Tensions in Modified String and Brane Theories 241
E.I. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva

IV. Integrable Systems

- Quantum Group in Roots of Unity and the Restriction of XXZ Model 253
A. Belavin
- Spaces of Boundary Values Related to a Multipoint Version of the KP -Hierarchy 263
G.F. Helminck
- Integrable Systems with Impurity 280
E. Ragoucy
- Two Exactly-Solvable Problems in One-Dimensional Hyperbolic Space 294
Č. Burdík and G.S. Pogosyan
- The Diagonal Affine Coset Construction of the Z_k Parafermion Hall States 301
L. Georgiev

V. Quantum Mechanical and Probabilistic Systems

- Measures on Spaces of Infinite-Dimensional Configurations, Group Representations, and Statistical Physics 313
G.A. Goldin, U. Moschella and T. Sakuraba
- On the n -Particle Wigner Quantum Oscillator: Noncommutative Coordinates and Particle Localisation 327
R.C. King, T.D. Palev, N.I. Stoilova and J. Van der Jeugt
- Bundle Gerbes and Brownian Motion 342
R. Léandre
- On a Spectral Triple Related to Discrete Quantum Mechanics 353
H.-D. Doebner and R. Matthes
- Path Integral Approach to Noncommutative Quantum Mechanics 364
B. Dragovich and Z. Rakić

VI. Quantum Groups and Related Objects

- Matrix Chain Models and Their q -Deformations 377
H.P. Jakobsen
- Exotic Bialgebras : Non-Deformation Quantum Groups 392
D. Arnaudon, A. Chakrabarti, V.K. Dobrev and S.G. Mihov
- The q -Boson-Fermion Realization of the Quantum Superalgebra $U_q(\mathfrak{gl}(m/n))$ 403
Č. Burdík and O. Navrátil
- Irreducible Representations of Liftings of Quantum Planes 414
N. Andruskiewitsch and M. Beattie

List of Participants

425

IMPACT OF DYNAMICAL TENSIONS IN MODIFIED STRING AND BRANE THEORIES

E.I. GUENDELMAN AND A. KAGANOVICH

*Department of Physics,
Ben-Gurion University, Beer-Sheva, Israel
email: guendel@bgumail.bgu.ac.il, alexk@bgumail.bgu.ac.il*

E. NISSIMOV AND S. PACHEVA

*Institute for Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia, Bulgaria
email: nissimov@inrne.bas.bg, svetlana@inrne.bas.bg*

First we briefly outline the general construction of Dp -brane models with *dynamical* tensions. We then proceed to a more detailed discussion of a modified string model where the string tension is related to the potential of (an external) world-sheet electric current. We show that cancellation of the pertinent conformal anomaly on the quantum level requires the dynamical string tension to be a square of a free massless world-sheet scalar field.

1. Main Motivation

Dirichlet p -branes (Dp -branes) [1] are $p+1$ -dimensional extended objects in space-time which carry the end points of fundamental open strings. Their crucial relevance in modern string theory is due to several basic properties of theirs such as providing explicit realization of non-perturbative string dualities, microscopic description of black-hole physics, gauge theory/gravity correspondence, large-radius compactifications of extra dimensions, brane-world scenarios in particle phenomenology, *etc.* . For a background on string and brane theories, see refs.[2].

In an independent recent development two of us have proposed a broad class of new models involving Gravity called *Two-Measure Gravitational Models* [3], whose actions are typically of the form:

$$S = \int d^D x \Phi(\varphi) L_1 + \int d^D x \sqrt{-g} L_2 , \quad (1)$$

$$L_{1,2} = e^{\frac{\alpha\phi}{M_P}} \left[-\frac{1}{\kappa} R(g, \Gamma) - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + (\text{Higgs}) + (\text{fermions}) \right] . \quad (2)$$

with the standard notations: $R(g, \Gamma)$ is the scalar curvature in the first-order formalism (*i.e.*, the connection Γ is independent of the metric $g_{\mu\nu}$), ϕ is the dilaton field, M_P is the Planck mass, *etc.*. The main new ingredient appears in the first term of (1) – it is an alternative non-Riemannian (*i.e.* independent of the metric $g_{\mu\nu}$) generally-covariant integration measure density $\Phi(\varphi)$ built up in terms of additional auxiliary scalar fields φ^i ($i = 1, \dots, D$ where D is the space-time dimension) :

$$\Phi(\varphi) \equiv \frac{1}{D!} \varepsilon^{\mu_1 \dots \mu_D} \varepsilon_{i_1 \dots i_D} \partial_{\mu_1} \varphi^{i_1} \dots \partial_{\mu_D} \varphi^{i_D} . \quad (3)$$

Although naively the additional “measure-density” scalars φ^i appear in (1) as pure-gauge degrees of freedom (due to the invariance under arbitrary diffeomorphisms in the φ^i -target space), there is still a remnant – the so called “geometric” field $\chi(x) \equiv \frac{\Phi(\varphi)}{\sqrt{-g}}$, which remains an additional dynamical degree of freedom beyond the standard physical degrees of freedom characteristic to the ordinary gravity models with the standard Riemannian-metric integration measure. The most important property of the “geometric” field $\chi(x)$ is that its dynamics is determined solely through the matter fields locally (*i.e.*, without gravitational interaction). The latter turns out to have a significant impact on the physical properties of the two-measure gravity models which allows them to address various basic problems in cosmology and particle physics phenomenology and provide physically plausible solutions, for instance: (i) the issue of scale invariance and its dynamical breakdown, *i.e.*, spontaneous generation of dimensionfull fundamental scales; (ii) cosmological constant problem; (iii) geometric origin of fermionic families. In the very recent papers [4] it has been demonstrated that two-measure gravity theories are of significant interest in the context of modern brane-world scenarios, namely, a new conformally invariant brane-world model in $D=6$ without (bulk) cosmological constant fine tuning has been constructed there.

Subsequently, the idea of employing alternative non-Riemannian reparametrization-covariant integration measures was applied in the context of strings and branes theories [5, 6]. A common basic property of these modified string/brane models is that the ratio of both integration measure densities (the alternative versus the standard Riemannian) becomes a dynamical string/brane tension – an additional dynamical degree of freedom beyond the original string/brane degrees of freedom. In particular, in ref.[6] we systematically constructed a new class of modified Dp -brane models with dynamical brane tension – this construction is briefly reviewed in Section 2. We have shown in [5, 6], that the dynamical nature of the string/brane

tension leads to some new interesting physical effects such as simple mechanisms of confinement of “color” point-like charges (in the string case) and of charged lower-dimensional sub-branes (in the Dp -brane case).

In Section 3 below we study in some detail the quantum properties of a modified string model where the dynamical tension becomes related to the potential of an external electric charge current on the string world-sheet. We explicitly show that quantum consistency, *i.e.*, cancellation of the pertinent conformal anomaly, apart from the well-known restriction on D ($D=26$ in the simplest bosonic case) implies that the dynamical string tension must be a square of a free massless world-sheet scalar field.

2. Dp -Branes with Dynamical Tension

First, let us recall the standard formulation of Dp -branes given in terms of the Dirac-Born-Infeld (DBI) action (see *e.g.* third ref.[2]) :

$$S_{DBI} = -T \int d^{p+1} \sigma \left[e^{-\alpha U} \sqrt{-\det \|G_{ab} - \mathcal{F}_{ab}\|} + \dots \right], \quad (4)$$

with the following short-hand notations:

$$G_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \quad , \quad \mathcal{F}_{ab} \equiv \partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X) - F_{ab}(A). \quad (5)$$

Here $G_{\mu\nu}(X)$, $U(X)$, and $B_{\mu\nu}(X)$ are the background metric, the dilaton, and the 2-form Neveu-Schwarz, respectively, whereas $F_{ab}(A) = \partial_a A_b - \partial_b A_a$ is the field-strength of the Abelian world-volume gauge field A_a . The dots in (4) indicate coupling to the $(p+1)$ -form Ramond-Ramond background gauge field which is omitted for simplicity. All world-volume indices take values $a, b = 0, 1, \dots, p$ and $\varepsilon^{a_1 \dots a_{p+1}}$ is the $(p+1)$ -dimensional totally antisymmetric tensor ($\varepsilon^{01 \dots p} = 1$).

Similarly to the gravity case (1)–(3) we now introduce a modified world-volume integration measure density in terms of $p+1$ auxiliary scalar fields φ^i ($i = 1, \dots, p+1$) :

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \varepsilon_{i_1 \dots i_{p+1}} \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} \varphi^{i_1} \dots \partial_{a_{p+1}} \varphi^{i_{p+1}}, \quad (6)$$

and use it to construct the following new p -brane-type action :

$$S = - \int d^{p+1} \sigma \Phi(\varphi) \left[e^{-\beta U} \frac{1}{2} \zeta^{ab} (G_{ba} - \mathcal{F}_{ba}) + \frac{1}{\sqrt{-\zeta}} \Omega(\mathcal{A}) \right] + \int d^{p+1} \sigma \mathcal{L}(\mathcal{A}). \quad (7)$$

Here apart from (5) the following new notations are used. The $(p+1) \times (p+1)$ matrix ζ_{ab} of auxiliary variables is an arbitrary world-volume 2-tensor, ζ^{ab} denotes the corresponding inverse matrix ($\zeta^{ac} \zeta_{cb} = \delta_b^a$) and

$\zeta \equiv \det \|\zeta_{ab}\|$. The term $\Omega(\mathcal{A})$ indicates a topological density given in terms of some additional auxiliary gauge fields \mathcal{A}^I living on the world-volume:

$$\frac{\partial \Omega}{\partial \mathcal{A}^I} - \partial_a \left(\frac{\partial \Omega}{\partial \partial_a \mathcal{A}^I} \right) = 0 \text{ identically} \quad , \quad \text{i.e. } \delta \Omega(\mathcal{A}) = \partial_a \left(\frac{\partial \Omega}{\partial \partial_a \mathcal{A}^I} \delta \mathcal{A}^I \right) . \quad (8)$$

Finally, $\mathcal{L}(\mathcal{A})$ describes possible coupling of the auxiliary fields \mathcal{A}^I to external ‘‘currents’’ on the brane world-volume.

The requirement for $\Omega(\mathcal{A})$ to be a topological density is dictated by the requirement that the new brane action (7) (in the absence of the last gauge-coupling term $\int d^{p+1} \sigma \mathcal{L}(\mathcal{A})$) reproduces the standard Dp -brane equations of motion resulting from the DBI action (4) apart from the fact that the Dp -brane tension $T \equiv \Phi(\varphi)/\sqrt{-\zeta}$ becomes now an *additional dynamical degree of freedom* (note that no *ad hoc* dimensionfull tension factor T has been introduced in (7)). Let us particularly stress that the modified-measure brane model (7) naturally requires (through the necessity to introduce topological density $\Omega(\mathcal{A})$) the existence on the world-volume of an additional (higher-rank tensor) gauge field \mathcal{A} apart from the standard world-volume Abelian vector gauge field A_a .

Splitting the auxiliary tensor variable $\zeta^{ab} = \gamma^{ab} + \zeta^{[ab]}$ into symmetric and anti-symmetric parts and setting $\zeta^{[ab]} = 0$, the action (7) reduces to the action of the modified-measure model of ordinary p -branes [5] with Neveu-Schwarz field $B_{\mu\nu}$ and world-volume gauge field A_a disappearing and γ_{ab} assuming the role of world-volume Riemannian metric.

The most obvious example of a topological density $\Omega(\mathcal{A})$ for the additional auxiliary world-volume gauge fields in (7) is:

$$\Omega(\mathcal{A}) = -\frac{\varepsilon^{a_1 \dots a_{p+1}}}{p+1} F_{a_1 \dots a_{p+1}}(\mathcal{A}) \quad , \quad F_{a_1 \dots a_{p+1}}(\mathcal{A}) = (p+1) \partial_{[a_1} \mathcal{A}_{a_2 \dots a_{p+1}]} , \quad (9)$$

where $\mathcal{A}_{a_1 \dots a_p}$ denotes rank p antisymmetric tensor (Abelian) gauge field on the world-volume. More generally we can have (for $p+1=r$):

$$\Omega(\mathcal{A}) = \frac{1}{rs} \varepsilon^{a_{11} \dots a_{1r} \dots a_{s1} \dots a_{sr}} F_{a_{11} \dots a_{1r}}(\mathcal{A}) \dots F_{a_{s1} \dots a_{sr}}(\mathcal{A}) \quad (10)$$

with rank $r-1$ (smaller than p) auxiliary world-volume gauge fields.

We may also employ *non-Abelian* auxiliary world-volume gauge fields \mathcal{A}_a . For instance, when $p+1=2q$:

$$\Omega(\mathcal{A}) = \frac{1}{2q} \varepsilon^{a_1 b_1 \dots a_q b_q} \text{Tr} (F_{a_1 b_1}(\mathcal{A}) \dots F_{a_q b_q}(\mathcal{A})) , \quad (11)$$

where $F_{ab}(\mathcal{A}) = \partial_a \mathcal{A}_b - \partial_b \mathcal{A}_a + i[\mathcal{A}_a, \mathcal{A}_b]$.

The equations of motion w.r.t. φ^i and ζ^{ab} corresponding to the modified-measure brane action (7) read:

$$e^{-\beta U} \frac{1}{2} \zeta^{ab} (G_{ba} - \mathcal{F}_{ba}) + \frac{1}{\sqrt{-\zeta}} \Omega(\mathcal{A}) = M \equiv \text{const} , \quad (12)$$

$$e^{-\beta U} (G_{ab} - \mathcal{F}_{ab}) + \zeta_{ab} \frac{1}{\sqrt{-\zeta}} \Omega(\mathcal{A}) = 0 . \quad (13)$$

Both Eqs.(12)–(13) imply:

$$\zeta^{ab} (G_{ba} - \mathcal{F}_{ba}) = 2M \frac{p+1}{p-1} e^{\beta U} , \quad \frac{1}{\sqrt{-\zeta}} \Omega(\mathcal{A}) = -\frac{2M}{p-1} \quad (14)$$

which when substituted in (13) give:

$$G_{ab} - \mathcal{F}_{ab} = \frac{2M}{p-1} e^{\beta U} \zeta_{ab} \quad (15)$$

Next we consider the equations of motion w.r.t. auxiliary (gauge) fields \mathcal{A}^I :

$$\partial_a \left(\frac{\Phi(\varphi)}{\sqrt{-\zeta}} \right) \frac{\partial \Omega}{\partial \partial_a \mathcal{A}^I} + j_I = 0 , \quad (16)$$

where $j_I \equiv \frac{\partial \mathcal{L}}{\partial \mathcal{A}^I} - \partial_a \left(\frac{\partial \mathcal{L}}{\partial \partial_a \mathcal{A}^I} \right)$ is the corresponding ‘‘current’’ coupled to \mathcal{A}^I . These are the equations determining the dynamical brane tension $T \equiv \Phi(\varphi)/\sqrt{-\zeta}$. In deriving Eq.(16) crucial use was made of the identity (8) satisfied by the topological density $\Omega(\mathcal{A})$.

In particular, in the absence of coupling of external world-volume currents to the auxiliary (gauge) fields \mathcal{A}^I Eq.(16) imply:

$$T \equiv \Phi(\varphi)/\sqrt{-\zeta} = C \equiv \text{const} \quad (17)$$

Now, using Eqs.(12) and (15) it is straightforward to show that the modified brane action (7) with $\mathcal{L}(\mathcal{A}) = 0$ classically reduces to the standard Dp -brane DBI-action (4) :

$$S'_{DBI} = -T' \int d^{p+1} \sigma e^{-\beta' U} \sqrt{-\det ||G_{ab} - \mathcal{F}_{ab}||} , \quad (18)$$

$$T' \equiv \frac{1}{2} C (2M)^{-\frac{p-1}{2}} (p-1)^{\frac{p+1}{2}} , \quad \beta' \equiv \frac{p+1}{2} \beta , \quad (19)$$

where, however, the Dp -brane tension T' is dynamically generated according to (17) and (19).

For a more detailed analysis of the properties of the modified-measure Dp -brane models we refer to [6].

3. Conformal Anomaly and Its Impact on the Dynamical String Tension

Now we turn our attention to a special case of (7) for $p=1$, i.e., a modified string model with dynamical tension:

$$S = - \int d^2\sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A) \right] + \epsilon \int d^2\sigma A_a \varepsilon^{ab} \partial_b u . \quad (20)$$

Notice that the last term in Eq.(20) can be rewritten in the reparametrization-invariant form:

$$\int d^2\sigma \sqrt{-\gamma} A_a J^a \quad , \quad J^a \equiv \epsilon \frac{\varepsilon^{ab}}{\sqrt{-\gamma}} \partial_b u , \quad (21)$$

where J^a indicates the general expression for a covariantly conserved world-sheet electric current. Let us stress that such coupling of string degrees of freedom to an external world-sheet electric current is natural only in the present context of modified-measure string models due to the inevitable appearance of the auxiliary world-sheet gauge field A_a . Let us also note that in (21) the ordinary Riemannian world-sheet integration measure density $\sqrt{-\gamma}$ is used unlike the modified one:

$$\Phi(\varphi) = \frac{1}{2} \varepsilon_{ij} \varepsilon^{ab} \partial_a \varphi^i \partial_b \varphi^j \quad , \quad a, b = 0, 1 \quad , \quad i, j = 1, 2 \quad (22)$$

in the main action term in (20), much in the spirit of the previously proposed two-measure gravity theories [3] (cf. Eq.(1)).

Quantization of the modified string model (20) within the functional integral framework can be performed in the standard way based on the canonical Hamiltonian formalism for constrained systems a'la Dirac. As already shown in the third ref.[5], the total canonical Hamiltonian $H_T \equiv \sum_A \Lambda_A \Phi_A$ is a linear combination of the following first-class constraints Φ_A (the letters π and \mathcal{P} indicating the pertinent canonical momenta) :

$$\pi_{\gamma^{ab}} = 0 \quad , \quad \mathcal{T}_\pm \equiv \frac{1}{4} \left(\frac{\mathcal{P}}{E} \pm \partial_\sigma X \right)^2 = 0 , \quad (23)$$

which are of the same form as in the ordinary bosonic string case modulo the fact that now the string tension $T \equiv E$ is a dynamical degree of freedom (see last Eq.(25) below), plus the new constraints:

$$\partial_\sigma \varphi^i \pi_i^\varphi = 0 \quad , \quad \frac{\pi_2^\varphi}{\partial_\sigma \varphi^1} = 0 ; \quad (24)$$

$$\pi_{A_0} = 0 \quad , \quad \partial_\sigma (E + \epsilon u) = 0 \quad , \quad \text{where } E \equiv \pi_{A_1} = \frac{\Phi(\varphi)}{\sqrt{-\gamma}} . \quad (25)$$

Relations (24) imply that the auxiliary measure-fields φ^i are pure-gauge degrees of freedom which decouple from the string dynamics. The only remnant of the latter appears through E – the canonical momentum of A_1 according to the last expression (25), which together with the second relation (25) tells us that E has the physical meaning of a world-sheet electric field-strength obeying the $D = 2$ Gauss law.

To quantize the modified-measure string model (20) one starts with the standard Faddeev's functional integral:

$$Z = \int \mathcal{D}X \mathcal{D}\mathcal{P} \mathcal{D}E \mathcal{D}A_1 \mathcal{D}\Lambda_A \Delta_{\Phi\Pi} \delta(\text{conf. gauge}) \\ \times \exp\left\{i \int d^2\sigma \left[\mathcal{P}_\mu \partial_\tau X^\mu + E \partial_\tau A_1 - \sum_A \Lambda_A \Phi_A \right]\right\} \quad (26)$$

where Φ_A are the first-class constraints listed above (23)–(25) and $\Delta_{\Phi\Pi}$ indicates the Faddeev-Popov ghost determinant associated with the conformal gauge-fixing condition. In (26) and in what follows we shall skip the insertion of vertex operators for brevity. Now performing the Gaussian integrations over the canonical momenta we arrive at the following reparametrization-invariant expression:

$$Z = \int \mathcal{D}X \mathcal{D}\gamma_{ab} \mathcal{D}E \mathcal{D}A_a \Delta_{\Phi\Pi} \delta(\text{conf. gauge}) \\ \times \exp\left\{i \int d^2\sigma \left[-E \frac{1}{2} \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu + \frac{1}{2} (E + \epsilon u) \varepsilon^{ab} F_{ab}(A) \right]\right\} \quad (27)$$

Integration over the auxiliary gauge field A_a yields functional delta-function $\delta(\varepsilon^{ab} \partial_b (E + \epsilon u))$ which in turn reduces the functional integration over E to an ordinary integration over the overall world-sheet constant C :

$$Z = \int dC \mathcal{D}X \mathcal{D}\gamma_{ab} \Delta_{\Phi\Pi} \delta(\text{conf. gauge}) \\ \times \exp\left\{-i \frac{1}{2} \int d^2\sigma (C - \epsilon u) \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu\right\} \quad (28)$$

Note that $T \equiv C - \epsilon u$ is the dynamical string tension. Thus, integration over string coordinates X^μ amounts to computation of the determinant of the modified D'Alembertian (or Laplace-Beltrami upon Euclidean rotation) operator:

$$-\frac{1}{\sqrt{-\gamma}} \partial_a (v^2 \sqrt{-\gamma} \gamma^{ab} \partial_b) \quad (29)$$

where for later convenience we have introduced the notation v for the square-root of the dynamical tension: $v^2 \equiv C - \epsilon u$. Due to the presence

of the latter there is an additional contribution to the well-known conformal anomaly (see e.g. [7] and references therein where the “dilaton”-like notation $v = e^{-\phi}$ is employed) :

$$Z = \int dC \mathcal{D}\gamma_{ab} \delta(\text{conf. gauge}) \exp \left\{ i \int d^2\sigma \sqrt{-\gamma} \left[\frac{D-26}{96\pi} R \square^{-1} R + \frac{D}{8\pi} \frac{\square v}{v} \square^{-1} R - b \frac{D}{4\pi} \gamma^{ab} (\nabla_a \ln v) (\nabla_b \ln v) \right] \right\} \quad (30)$$

Here R denotes the usual scalar curvature for the intrinsic world-sheet metric γ_{ab} and $\square \equiv \frac{1}{\sqrt{-\gamma}} \partial_a (\sqrt{-\gamma} \gamma^{ab} \partial_b)$ is the ordinary covariant D'Alembertian. The last non-anomalous term in (30) is determined up to a regularization-dependent constant b . Therefore, absence of conformal anomaly implies in the present case, apart from the usual critical value for the space-time dimension, an additional condition on the square-root of the dynamical string tension:

$$\square v = 0 \quad , \quad v^2 \equiv T = C - \epsilon u \quad , \quad (31)$$

i.e., the dynamical string tension must be a square of a free massless world-sheet scalar field. Note that this condition is a dynamical constraint on the external world-sheet electric current (21) coupled to the modified-measure string.

Going back to the string action in (28) and taking into account relations (31) we can rewrite it in the following simple form:

$$-\frac{1}{2} \int d^2\sigma \sqrt{-\gamma} \gamma^{ab} \partial_a (v X^\mu) \partial_b (v X_\mu) \quad (32)$$

where we used the following identity for the modified D'Alembertian operator (29) :

$$\frac{1}{\sqrt{-\gamma}} \partial_a (v^2 \sqrt{-\gamma} \gamma^{ab} \partial_b) (\cdot) = v \square (v \cdot) - (\square v) v = v \square (v \cdot) \quad (33)$$

due to the free masslessness of the square-root tension v . Therefore, in the quantized modified-measure string model (20) it is the dynamically rescaled fields $\tilde{X}^\mu = v X^\mu$ which describe free wave mode propagation along the string rather than the usual string coordinates X^μ .

4. Conclusions

Replacing the standard Riemannian world-sheet/world-volume integration measure density with an metric-independent reparametrization-invariant one (6) in the Lagrangian formulation of string and brane models has

significant impact on the string/brane dynamics. Consistency of dynamics requires the introduction of additional auxiliary (higher-rank) gauge fields on the world-sheet/world-volume which are absent in the standard string/brane theories. The main new property of the modified-measure string/brane models is that the string/brane tension appears as an additional dynamical degree of freedom which is canonically conjugated to the auxiliary world-sheet/world-volume gauge fields. It acquires the physical meaning of world-sheet electric field-strength (in the string case) or field-strength of higher-rank world-volume gauge fields (in the brane case) obeying the Maxwell (or Yang-Mills) equations of motion or their higher-rank generalizations. As a simple consequence of the latter, modified-measure string/brane models provide (already on the classical level) simple mechanisms for “color” charge confinement. Furthermore, in the quantized string context the interplay between the intrinsic conformal anomaly and the dynamical nature of the string tension imply an important constraint on the form of the dynamical string tension forcing it to be a square of a free massless world-sheet scalar. It is curious to note that the last property resembles the property found in the context of the string-inspired low-energy effective field theory in $D = 10$ [8] where the square of the target-space dilaton field plays the role of a covariant integration measure density

More detailed study of the effects resulting from the new physical properties of the modified-measure string models with dynamical tension reported above will be done in a separate work.

Acknowledgments

Two of us (E.N. and S.P.) are sincerely grateful to the organizers of the Fifth Workshop on *Lie Theory and Applications in Physics* (Varna, Bulgaria) for the kind invitation to present there the above results. E.N. and S.P. also acknowledge cordial hospitality and illuminating discussions with Prof. Yannis Bakas at the University of Patras (Greece). The work of E.N. and S.P. is partially supported by NATO collaborative linkage grant *PST.CLG.978785* and Bulgarian NSF grant *F-904/99*. Finally, all of us acknowledge support of our collaboration through the exchange agreement between the Ben-Gurion University of the Negev and the Bulgarian Academy of Sciences.

References

1. Polchinski, J. (1996), “*TASI Lectures on D-Branes*” (hep-th/9611050).

2. Ne'eman, Y. and Eizenberg, E. (1995), “*Membranes and Other Extensions*”, World Scientific;
Green, M., Schwarz, J. and Witten, E. (1987), “*Superstring Theory*”, Vol.1,2, Cambridge Univ. Press;
Polchinski, J. (1998), “*String Theory*”, Vol.1,2, Cambridge Univ. Press.;
Johnson, C. (2002), “*D-Branes*”, Cambridge Univ. Press, and references therein (see also hep-th/0007170).
3. Guendelman, E.I. and Kaganovich, A. (1996), *Phys. Rev.* **D53** 7020;
Guendelman, E.I. and Kaganovich, A. (1999), *Phys. Rev.* **D60** 065004;
Guendelman, E.I.(1999), *Mod. Phys. Lett.* **A14** 1043;
Guendelman, E.I. (2000), *Class. Quant. Grav.* **17** 261;
Kaganovich, A. (2001), *Phys. Rev.* **D63** 025022;
Guendelman E.I. and Kaganovich, A. (2002), *Int. J. Mod. Phys.* **A17** 417 (hep-th/0106152);
Guendelman E.I. and Kaganovich, A. (2002), *Mod. Phys. Lett.* **A17** 1227 (hep-th/0110221);
Guendelman E.I. and Kaganovich, A. (2003), gr-qc/0312006.
4. Guendelman, E.I. (1997), *Phys. Lett.* **412B** 42;
Guendelman, E.I. (2003), gr-qc/0303048;
Guendelman, E.I. and Spallucci, E. (2003), hep-th/0311102.
5. Guendelman, E.I. (2000), *Class. Quantum Grav.* **17** 3673 (hep-th/0005041);
Guendelman, E.I. (2001), *Phys. Rev.* **D63** 046006 (hep-th/0006079);
Guendelman, E.I., Kaganovich, A., Nissimov, E. and Pacheva, S. (2002), *Phys. Rev.* **D66** 046003 (hep-th/0203024);
Guendelman, E.I., Kaganovich, A., Nissimov, E. and Pacheva, S. (2003), in “*First Workshop on Gravity, Astrophysics and Strings*”, P. Fiziev et.al. eds., Sofia Univ. Press, 2003 (hep-th/0210062).
6. Guendelman, E.I., Kaganovich, A., Nissimov, E. and Pacheva, S. (2003), in the proc. of the “*Second Internat. School on Modern Math. Physics*”, Kopaonik (Serbia and Montenegro), B. Dragovich et.al. eds. (2003) (hep-th/0304269).
7. Grumiller, D., Kummer W. and Vassilevich, D. (2002), *Phys. Reports* **369** 327 (hep-th/024253);
Vassilevich, D. (2003), hep-th/0306138.
8. Siegel, W. (1993), *Phys. Rev.* **D47** 5453 (hep-th/9302036);
Siegel, W. (1993), *Phys. Rev.* **D48** 2826 (hep-th/9305073).